Teacher notes
Topic D

Is the Earth moving away from the Sun?

The Sun loses mass because it converts mass into the energy it radiates. It also loses mass because of solar winds which carry mass away from the Sun at a rate of about $1.27\times10^{9}\ \text{kg s}^{-1}$.

What is the effect of this mass loss on the Earth's orbit?

We know that (M = Sun mass, m = Earth mass, r = Earth orbit radius = 1.5×10^{11} m)

$$\frac{mv^2}{r}=\frac{GMm}{r^2}\Rightarrow v^2=\frac{GM}{r}$$

The angular momentum of the Earth around the Sun is constant and equals $mvr=k$ (k = constant). Hence $vr=k'$ (another constant). This means that $v=\frac{k'}{r}$ and so

$$\frac{k'^2}{r^2}=\frac{GM}{r}$$ i.e. $Mr=\text{constant}$ or $r=\frac{\text{constant}}{M}$.

Using ideas of uncertainty propagation we find:

$$\frac{\Delta r}{r}=-\frac{\Delta M}{M}$$

So, the change in the orbit radius is determined by the fractional change in the mass of the Sun. This fractional change is negative because the Sun loses mass. Hence the radius increases; the Earth is moving away from the Sun.

In 1 second the Sun loses 3.8×10^{26} J and so a mass of $\frac{3.8\times10^{26}}{(3.0\times10^{8})^2}=4.22\times10^{9}\ \text{kg s}^{-1}$. The mass lost due to the solar wind is $1.27\times10^{9}\ \text{kg s}^{-1}$ for a total rate of loss of mass of $5.49\times10^{9}\ \text{kg s}^{-1}$. In a billion years this amounts to a mass of $5.49\times10^{9}\times10^{9}\times365\times24\times60\times60=1.73\times10^{26}\ \text{kg}$.

Hence $\frac{\Delta r}{r}=\frac{1.73\times10^{26}}{2.0\times10^{30}}=8.65\times10^{-5}$ giving $\Delta r=1.5\times10^{11}\times8.65\times10^{-5}\approx1.3\times10^{7}\ \text{m}$. This amounts to $\frac{1.3\times10^{7}}{10^{9}}=1.3\times10^{-2}$ m per year or 1.3 cm per year. (This is of no use in solving the climate crisis!)